Learning Objective

The purpose of this project is to practice modeling a practical dynamical system, designing control strategies with specific sensory inputs, analyzing the system performance, and implementing a Kalman filter for sensor fusion.

Report Requirement

On-campus students must hand in the reports in paper copy. EDGE students may upload the work via Sakai. Please include all the figures and MATLAB codes in your report, together with necessary theoretical derivations and discussions. Figures (and possibly tables) should be visibly titled and labeled with proper font size. Make sure that the program codes are well commented for easy understanding. The report should not exceed 10 pages. However, the limit is only intended to reduce your workload and can be relaxed. All reports should be submitted before the deadline with no exception.

Background

An autonomous underwater vehicle (AUV) is a robot vehicle which travels underwater without requiring input from an operator. It can be used for performing dangerous tasks in hazardous locations or surveying vast areas for underwater exploration and environmental monitoring.

The AUV model in this project will be based on the prototype CephaloBot developed in the group [1]. As shown in Figure 1 and Figure 2, the vehicle is equipped with cephalopod inspired vortex ring thrusters (VRTs) that can provide quantized propulsive force by creating arrays of high-momentum vortex rings with successive ingestion and expulsion of water [2, 3]. Compared to the control surfaces used in conventional torpedo-shaped AUVs and the multiple thrusters used in box-shaped AUVs, this device allows the vehicle to perform accurate maneuver at low speeds, without sacrificing its low-drag streamline profile for efficient high-speed traveling [4, 5, 6].

Project Problems

In this project, a nonlinear dynamic system of the AUV will be introduced, based on which you will be designing controllers to regulate the vehicle’s horizontal movement.

Assume the earth-fixed frame to be inertial. The position and orientation of the vehicle is described relative to the inertial frame while the linear and angular velocities is expressed with respect to the body-fixed frame. Since the movement is restricted inside the horizontal plane, there are three degrees of freedom in the dynamics, namely, translational motions along X and Y directions (surge and sway), and rotational motion about Z axis (yaw).

The vehicle’s linear velocities in X and Y directions and angular velocity about Z axis with respect to the body-fixed frame is designated as signal \( \mathbf{\nu}(t) = [\nu_1(t) \ \nu_2(t) \ \nu_3(t)]^\top \in \mathbb{R}^3 \). The velocity vector of background
flow is defined as \( \nu_f(t) = [\nu_{f1}(t) \ \nu_{f2}(t) \ \nu_{f3}(t)]^\top \in \mathbb{R}^3 \). The vehicle dynamics can be written in the form of a continuous-time nonlinear system as

\[
B_M \tau = M \dot{\nu} + C_M(\nu) \nu + D_M(\nu_f - \nu)(\nu_f - \nu),
\]

where \( \tau(t) = [\tau_1(t) \ \tau_2(t) \ \tau_3(t)]^\top \in \mathbb{R}^3 \) denotes the control command to the actuators. The matrix \( B_M \in \mathbb{R}^{3\times 3} \) denotes the actuator function, \( M \in \mathbb{R}^{3\times 3} \) is the inertia matrix of the vehicle, \( C_M(\nu) \in \mathbb{R}^{3\times 3} \) represents the Centripetal-Coriolis matrix, and \( D_M(\nu_f) \in \mathbb{R}^{3\times 3} \) denotes the matrix of drag coefficients. The matrices are defined by

\[
B_M = \begin{bmatrix} b_1 & 0 & 0 \\ 0 & b_2 & b_2 \\ 0 & b_3 & -b_3 \end{bmatrix}, \quad M = \begin{bmatrix} p_1 & 0 & 0 \\ 0 & p_1 & p_2 \\ 0 & p_2 & p_3 \end{bmatrix}, \quad C_M(\nu) = \begin{bmatrix} 0 & 0 & -p_1 \nu_2 - p_2 \nu_3 \\ 0 & 0 & p_1 \nu_1 \\ p_1 \nu_2 + p_2 \nu_3 & -p_1 \nu_1 & 0 \end{bmatrix},
\]

and \( D_M(\nu_f - \nu) \) is a diagonal matrix \( D_M(\nu_f - \nu) = \text{diag}(-d_1 \nu_{f1}, -d_2 \nu_{f2}, -d_3 \nu_{f3}) \). The parameters in the matrices are defined to be

\[
b_1 = 1 \text{ N}, \quad p_1 = 20 \text{ kg}, \quad d_1 = 6 \text{ kg} \cdot \text{s/m}^2, \\
b_2 = 0.5 \text{ N}, \quad p_2 = 2 \text{ kg} \cdot \text{m}, \quad d_2 = 80 \text{ kg} \cdot \text{s/m}^2, \\
b_3 = 0.2 \text{ N} \cdot \text{m}, \quad p_3 = 4 \text{ kg} \cdot \text{m}^2, \quad d_3 = 7 \text{ kg} \cdot \text{m}^2 \cdot \text{s}.
\]

Complete the following analysis and design tasks.

(a) As implied by the vehicle model, the hydrodynamic drag on the vehicle \( D_M(\nu_f - \nu) \) is a function of the relative velocity of the vehicle with respect to the flow. If we assume that the vehicle is operating in an ambient environment so that \( \nu_f \equiv 0 \), the state of the system can be defined as \( \nu(t) \), and \( \tau(t) \) can be treated as the input signal. Suppose the vehicle is moving in a horizontal circle with a radius of 10 m at a constant speed of 0.5 m/s. Thus, the pair \( (\nu_{eq}, \tau_{eq}) \in \mathbb{R}^3 \times \mathbb{R}^3 \) forms an equilibrium point, in which

\[
\nu_{eq} = \begin{bmatrix} 0.5 \text{ m/s} \\ 0 \text{ m/s} \\ 0.05 \text{ rad/s} \end{bmatrix}.
\]

• Find the corresponding input signal \( \tau_{eq} \) of the equilibrium.
• Locally linearize the system around the equilibrium point \((\nu^e, \tau^e)\).

• Express the linearized system in the standard form of the state equation

\[
\dot{x}(t) = A \ x(t) + B \ u(t),
\]

(5)

where the signals \(x(t) \in \mathbb{R}^3\) and \(u(t) \in \mathbb{R}^3\) are perturbation values of the original system with respect to \(\nu^e\) and \(\tau^e\):

\[
x(t) = \delta\nu(t) = \nu(t) - \nu^e, \quad u(t) = \delta\tau(t) = \tau(t) - \tau^e.
\]

(6)

• Suppose that the vehicle is operating in a wavy environment such that \(\nu_f\) is not always zero, and onboard the vehicle there is a flow velocity sensor that provides measurements of the flow velocity with respect to the vehicle \(\nu_f - \nu\). Therefore, we are able to obtain the signal \(\nu_f\). Based on (1), which term in the equation will be different if we make the assumption \(\nu_f \neq 0\) rather than \(\nu_f \equiv 0\)? Obviously, we can apply an additional control command signal \(\tau_0\) to compensate for the differences. From (1), what will be the control command \(\tau_0\)?

(b) Consider the continuous-time homogeneous LTI system with \(u(t) = 0\):

\[
\dot{x}(t) = A \ x(t).
\]

(7)

• Classify the system in terms of its Lyapunov stability (i.e., marginally stable, asymptotically stable, exponentially stable, or unstable).

• Suppose the vehicle is operating at prescribed velocity state \(\nu(t) = \nu^e\) with a constant control input \(\tau(t) = \tau^e\), when an unknown disturbance from the water strikes and deviates its velocity state by \(\Delta\nu\). Based on your result, is it guaranteed that the vehicle will automatically get back to the designated velocity after the disturbance?

(c) In the following problems, you will be asked to design controllers for the actuation tests on one of the VRTs onboard the vehicle. The test is to compare control command \(u(t) \in \mathbb{R}\) of the VRT and the angular measurements \(y(t) \in \mathbb{R}\) from the magnetic reference sensors. Assume that the linearized state equation and the output equation can be expressed as

\[
\dot{x}(t) = A \ x(t) + B \ u(t), \quad y(t) = C \ x(t),
\]

(8)

where

\[
A = \begin{bmatrix} -0.1 & -20 \\ 0.2 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 10 \\ 0 \end{bmatrix}, \quad C = [0.2 \ 2].
\]

(9)

• Plot the unit step response, the Bode plot, and the Nyquist plot for the given open-loop system with MATLAB®.

• Determine the controllability of the given system.

• Assume all the states are measurable. Design a state feedback controller \(u(t) = -K \ x(t) + r(t)\), such that the resultant system

\[
\dot{x}(t) = (A - BK) \ x(t) + B \ r(t), \quad y(t) = C \ x(t),
\]

(10)

is asymptotically stable; and with a rise time of \(T_r\), a settling time \(T_s\) and an overshoot \(OS\%\):

\[
T_r \leq 0.1 \ \text{s}, \quad T_s \leq 1.2 \ \text{s}, \quad OS\% \leq 20\%.
\]

(11)
- Plot the unit step response, the Bode plot, and the Nyquist plot for the state feedback system. Discuss the gain margin and phase margin of the closed-loop system.

- Compare the performance of state feedback system to that of the original system.

(d) Based on your result from (c), complete the following analysis and design:

- Determine the observability of the system in (8).

- Assume that only the output is measurable. Use the same state feedback gain \( K \) as in (c) to design an observer-based feedback controller such that the eigenvalues of the state estimator matrix \( (A - LC) \) lie to the left of \(-5\) on the left-hand half plane.

- Discuss the stability of the observer-based output feedback system.

Note: Use the Lyapunov-based analysis instead of the place command to solve for the feedback gain. The observability Gramian can be obtained with the lyap command in MATLAB®.

(e) Design two LQR controllers for the system in (8) by determining the optimal state feedback gain \( K \) with the following two sets of parameters respectively:

(i) \[ Q = \begin{bmatrix} 100 & 0 \\ 0 & 0 \end{bmatrix}, \quad R = 50; \]

(ii) \[ Q = \begin{bmatrix} 50 & 0 \\ 0 & 0 \end{bmatrix}, \quad R = 100. \]

(12) Comment on the performance differences between the two closed-loop systems.

(e) In this problem, you will combine noisy sensor measurements to estimate the yaw angle of an AUV. The yaw angle, \( \psi \), and the yaw rate, \( r \), are defined in figure 3. Suppose that the true yaw angle follows a sinusoid, i.e.

\[ \psi = A \sin \left( \frac{2\pi t}{T} \right), \]  

where \( A \) is the amplitude and \( T \) is the period. Furthermore, suppose that your sensors are able to measure the yaw angle and the yaw rate. Figure 4a shows the yaw angle measurement. The noise in the signal has a standard deviation of \( \sigma = 10^\circ \). Figure 4b shows the yaw rate measurement. While the noise in the signal is smaller (\( \sigma = 0.02^\circ/s \)), the bias drifts over time.

Combining noisy sensor measurements is typically done with a Kalman filter. The Kalman filter equations are summarized in (14) below. Table 1 defines all the symbols in (14).
\[ \dot{x}_{k|k-1} = F \dot{x}_{k-1|k-1} + B u_k \]

\[ P_{k|k-1} = FP_{k-1|k-1}F^T + Q_k \]

\[ K_k = P_{k|k-1}H^T(HP_{k|k-1}H^T + R_k)^{-1} \]

\[ \dot{x}_{k|k} = \dot{x}_{k|k-1} + K_k(z_k - H \dot{x}_{k|k-1}) \]

\[ P_{k|k} = P_{k|k-1} - K_kHP_{k|k-1} \]

(14)

**Table 1: Symbols in the Kalman filter**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \dot{x} )</td>
<td>State vector of a linear dynamic system.</td>
</tr>
<tr>
<td>( u )</td>
<td>Vector of control inputs</td>
</tr>
<tr>
<td>( z )</td>
<td>Vector of measured values</td>
</tr>
<tr>
<td>( F )</td>
<td>Matrix defining the linear dynamic system</td>
</tr>
<tr>
<td>( B )</td>
<td>Control input matrix</td>
</tr>
<tr>
<td>( H )</td>
<td>Measurement sensitivity matrix</td>
</tr>
<tr>
<td>( P )</td>
<td>Covariance matrix of state estimation uncertainty</td>
</tr>
<tr>
<td>( Q )</td>
<td>Covariance matrix of process noise</td>
</tr>
<tr>
<td>( R )</td>
<td>Covariance matrix of measurement uncertainty</td>
</tr>
<tr>
<td>( K )</td>
<td>Kalman gain matrix.</td>
</tr>
</tbody>
</table>

Assume that the yaw rate is the system input, \( u \). Also assume the yaw angle is the system measurement, \( z \). For this problem, you will compare two different methods of setting up the state-space model of the system. In the first method, the model will implement Euler integration:

\[ \psi_k = \psi_{k-1} + (\Delta t)r_k. \]

(15)

In the second method, the model will explicitly take into account the gyro bias drift, \( d \). That is, \( d \) will be a state of the system. In this case, the model is given as
\[
\begin{align*}
\psi_k &= \psi_{k-1} + (\Delta t)(r_k - d_{k-1}), \\
d_k &= d_{k-1}.
\end{align*}
\] (16)

The objective of this problem is to design a Kalman filter that combines the magnetometer measurement and the gyroscope measurement to obtain an estimate the yaw angle. You should find that the second method is less susceptible to drift than the first method. To help you complete this problem, an incomplete MATLAB code is provided to you; your job will be to complete the code. Specifically, you should

- Determine the \( F \) and \( B \) matrices corresponding to (15) and (16). Implement the matrices in the MATLAB code.
- Implement the Kalman filter equations given in (14) in the MATLAB code.
- Using the first implementation, plot a time history of the true yaw angle and the estimated yaw angle on the same graph.
- Using the second implementation, plot a time history of the true yaw angle and the estimated yaw angle on the same graph.
- Comment on whether the second implementation reduces drift.

References


