Learning Objective

The purpose of this project is to practice modeling a practical dynamical system, designing control strategies with specific sensory inputs, analyzing the system performance, and implementing a Kalman filter for sensor fusion.

Report Requirement

On-campus students need to submit their reports in hardcopy and EDGE students can submit their work on Sakai electronically. Please include all your figures and MATLAB codes besides of your theoretical derivation and discussion. Figures should have visible titles and axis labels in proper font sizes and please make sure your codes are well commented for easy understanding. We do not expect your whole report to exceed 10 pages. It’s not a strict limit but meant to reduce your workload. But if you did, we would love to read your report. All reports should be submit by the deadline and no exception will be accepted.

Background

Unmanned aerial vehicles (UAVs) are aircrafts without human pilots on board. They are often preferred for the missions that are too "dull, dirty, or dangerous" for manned aircraft such as military intelligence reconnaissance, civil application like firefighting, environmental surveillance, and weather forecasting. Currently, direct measurements of tropical storms and hurricanes are taken primarily by reconnaissance aircraft, although ships and buoys also take important measurements. NASA’s unmanned Global Hawk drone completed a 26-hour flight gathering information on Tropical Storm Nadine. Due to the low cost and no threats to human lives, the small sized UAVs is a promising topic.

In our group, we mainly study on the delta wing UAV (figure. 2a) and the micro aerial vehicle (MAV) (figure. 2b). Our delta wing UAVs are low cost platforms used in the applications like data collection, environmental monitoring, weather prediction, urban hazard plume source positioning, etc. Due to the on-board sensor and computation limitations, the study of delta wing UAVs focuses cooperation among wireless sensor network of UAVs [1, 2, 3]. The dynamics of the MAVs in small sizes are not well understood due to the unique body shape and aspect ratio. So our focus on MAV is the development of the governing dynamical equations of these vehicles [4, 5, 6].

Project Problems

In this project, you are asked to start with the general aircrafts’ equations of of motion, use your knowledge learn in this course and previous undergraduate control courses, and perform some analysis and designing to meet some particular requirements. Please include all your calculation steps in the report. All your figures should have correct labels and titles and please make sure they are in proper font sizes so that they are visible on A4 paper. Adding discussions after each figure is a good way to make your point clear and keep your report concise. Include all your m. files in your report comment your code properly.
Starting at the a time when we know the airplane’s position, orientation, translational velocity, rotational velocity, airspeed, aerodynamic angles, and angular velocity components, we can compute the aerodynamic forces and moments, including thrust. We can then compute the time derivatives from the Newton’s second law and the kinematic transformation equations. Assume the aircraft is operating in steady flight, we want to study the effect of given factors on the airplane in the longitudinal direction. The longitudinal equations of motion for the aircraft can be reduced by setting all the lateral disturbances to be zeros and to get the form

$$
\begin{bmatrix}
W/g & 0 & 0 \\
0 & W/g & 0 \\
0 & 0 & I_{yyb}
\end{bmatrix}
\begin{bmatrix}
\dot{u} \\
\dot{w} \\
\dot{q}
\end{bmatrix}
= 
\begin{bmatrix}
F_{xb} + W_{xb} - quW/g \\
F_{zb} + W_{zb} - quW/g \\
M_{yb}
\end{bmatrix},
$$

(1)

where $W$, $g$, $I_{yyb}$ are constants; $F_{xb}$, $F_{zb}$ are the functions of $\dot{u}$, $\dot{w}$, $u$, $w$, $q$, $u_f$, $w_f$, $q_f$, and $\delta_e$; $M_{yb}$ is a function of $\dot{u}$, $\dot{w}$, $u$, $w$, $q$, and $\delta_e$ and

$$
W_{xb} = -\sin(\theta) \quad \text{and} \quad W_{zb} = \cos(\theta).
$$

(2)

The variables $u_f$, $w_f$, and $q_f$ denote the relative air flow velocity with respect to the ground.
• Assuming that the air is ambient such that \( u_f = w_f = q_f \equiv 0 \), we can define the state vector and the control input vector as

\[
\begin{bmatrix}
u \\
w \\
q \\
\delta \\
\theta 
\end{bmatrix}
\quad \text{and} \quad
\begin{bmatrix}
\delta_e \\
\theta
\end{bmatrix}
\]

Linearize the longitudinal equations to the standard state-space form

\[
\dot{x} = Ax + Bu,
\]

around the trim condition (equilibrium state)

\[
x_{eq} = \begin{bmatrix} V_0 \\ 0 \\ 0 \end{bmatrix} \quad \text{and} \quad u_{eq} = \begin{bmatrix} 0 \\ \theta_0 \end{bmatrix},
\]

( Note: you only need to calculate the expression for matrices \( A \) and \( B \), e.g. you can simply express \( \frac{\partial F_{z_b}}{\partial \dot{u}} \) as \( F_{z_b,\dot{u}} \) )

• Generally, the relative air-to-ground speed is non-zero, so the relative air flow velocities are another ‘state’ of the dynamical system. Define another state variable as

\[
z = \begin{bmatrix} u_f \\ w_f \\ q_f \end{bmatrix}.
\]

Now, the right-hand side of the dynamical system in (1) can be linearized with respect to the variable \( z \) at equilibrium state \( z_{eq} \). Add this additional term into the state equation (4) so that we have a new state equation

\[
\dot{x} = Ax + Bu + Dz.
\]

Suppose there exist an air flow velocity sensor on-board the vehicle and it provides the velocity measurement \( u_f - u, w_f - w, \) and \( q_f - q \). Since the vehicle velocities \( u, w, \) and \( q \) are measurable, we can safely assume that the variables \( u_f, w_f, \) and \( q_f \) are known. If we would like to reduce the dynamical equation back to (4) by canceling the term \( Dz \) with \( Bu_0 (Bu_0 + Dz = 0) \), what should be the control input \( u_0 \)?

(b) Now consider an autopilot designing problem. The linearized equations can be expressed in state-space form as

\[
\begin{align*}
\dot{x}(t) &= Ax(t) + Bu(t) \\
y(t) &= Cx(t)
\end{align*}
\]

where

\[
A = \begin{bmatrix} -0.1 & -2 \\ 2 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 2 \\ 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 1 \end{bmatrix}.
\]

• Plot the unit step response, Bode plot, and Nyquist plot of the given open-loop system in MATLAB®.

• Determine the controllability of the given system.

• Assume all the states are measurable, design a state feedback controller \( u(t) = -Kx(t) + r(t) \), where \( r(t) = 1 \), such that the resultant system is asymptotically stable

\[
\begin{align*}
\dot{x}(t) &= (A - BK)x(t) + Br(t) \\
y(t) &= Cx(t)
\end{align*}
\]

and at the same time, meet the following requirements on rise time \( T_r \), the settling time \( T_s \) and the overshoot \( OS\% \):

\[
T_r \leq 0.1 \text{ s}, \quad T_s \leq 1.2 \text{ s}, \quad \text{OS\%} \leq 20\%.
\]
• Plot the unit step response, Bode plot, and Nyquist plot of the closed-loop system. Discuss the gain margin and phase margin.

• Compare the system with state feedback controller with the original system.

(c) Continue with (b) and perform the following analysis and designing:

• Determine the observability of the system defined by (8).

• Assuming that only the output is measurable, use the same state feedback gain $K$ as in (b) to design an observer-based feedback controller such that the eigenvalues of the state estimator matrix $(A - LC)$ lie to the left of $-5$ on the left-hand half plane.

• Explain how you can make sure the overall feedback system is asymptotically stable. (Hint: You are required to use the Lyapunov-based analysis method instead of the "place" command to solve this problem but you can use the "lyap" command in MATLAB® to solve for the observability Gramian.)

(d) Design two state-feedback LQR controllers for the system defined by (8) by finding the optimal state feedback gain $K$ in the state-feedback law $u = -Kx$ with

- $Q = \begin{bmatrix} 100 & 0 \\ 0 & 0 \end{bmatrix}$ and $R = 50$.

- $Q = \begin{bmatrix} 50 & 0 \\ 0 & 0 \end{bmatrix}$ and $R = 100$.

Discuss the difference of this two controllers.

(e) In this problem, you will combine noisy sensor measurements to estimate the roll angle of an aircraft. The roll angle, $\phi$, and the roll rate, $p$, are defined in figure 3. Suppose that the true roll angle follows a sinusoid, i.e.

$$\phi = A \sin \left( \frac{2\pi t}{T} \right), \quad (12)$$

where $A$ is the amplitude and $T$ is the period. Furthermore, suppose that your sensors are able to measure the roll angle and the roll rate. Figure 4a shows the roll angle measurement. The noise in the signal has a standard deviation of $\sigma = 10^\circ$. Figure 4b shows the roll rate measurement. While the noise in the signal is smaller ($\sigma = 0.02^\circ/s$), the bias drifts over time.

Figure 3: Defines the roll angle, $\phi$, and the roll rate, $p$, of the aircraft (assumes the aircraft’s nose is pointing out of the page). The inertial coordinate system is denoted with the $i$ subscript; the body coordinate system is denoted with the $b$ subscript.
Combining noisy sensor measurements is typically done with a Kalman filter. The Kalman filter equations are summarized in (13) below. Table 1 defines all the symbols in (13).

\[
\begin{align*}
\dot{x}_{k|k-1} &= F\dot{x}_{k-1|k-1} + Bu_k \\
P_{k|k-1} &= FP_{k-1|k-1}F^T + Q_k \\
K_k &= P_{k|k-1}H^T(HP_{k|k-1}H^T + R_k)^{-1} \\
\dot{x}_{k|k} &= \dot{x}_{k|k-1} + K_k(z_k - H\dot{x}_{k|k-1}) \\
P_{k|k} &= P_{k|k-1} - K_kHP_{k|k-1}
\end{align*}
\]

(13)

Table 1: Symbols in the Kalman filter

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\dot{x})</td>
<td>State vector of a linear dynamic system.</td>
</tr>
<tr>
<td>(u)</td>
<td>Vector of control inputs</td>
</tr>
<tr>
<td>(z)</td>
<td>Vector of measured values</td>
</tr>
<tr>
<td>(F)</td>
<td>Matrix defining the linear dynamic system</td>
</tr>
<tr>
<td>(B)</td>
<td>Control input matrix</td>
</tr>
<tr>
<td>(H)</td>
<td>Measurement sensitivity matrix</td>
</tr>
<tr>
<td>(P)</td>
<td>Covariance matrix of state estimation uncertainty</td>
</tr>
<tr>
<td>(Q)</td>
<td>Covariance matrix of process noise</td>
</tr>
<tr>
<td>(R)</td>
<td>Covariance matrix of measurement uncertainty</td>
</tr>
<tr>
<td>(K)</td>
<td>Kalman gain matrix.</td>
</tr>
</tbody>
</table>

Assume that the roll rate is the system input, \(u\). Also assume the roll angle is the system measurement, \(z\). For this problem, you will compare two different methods of setting up the state-space model of the system. In the first method, the model will implement Euler integration:

\[
\phi_k = \phi_{k-1} + (\Delta t)p_k. 
\]

(14)
In the second method, the model will explicitly take into account the gyro bias drift, \( d \). That is, \( d \) will be a state of the system. In this case, the model is given as

\[
\phi_k = \phi_{k-1} + (\Delta t)(p_k - d_{k-1}),
\]

\[
d_k = d_{k-1}.
\]

The objective of this problem is to design a Kalman filter that combines the accelerometer measurement and the gyroscope measurement to obtain an estimate the roll angle. You should find that the second method is less susceptible to drift than the first method. To help you complete this problem, an incomplete MATLAB code is provided to you; your job will be to complete the code. Specifically, you should

- Determine the \( \mathbf{F} \) and \( \mathbf{B} \) matrices corresponding to (14) and (15). Implement the matrices in the MATLAB code.
- Implement the Kalman filter equations given in (13) in the MATLAB code.
- Using the first implementation, plot a time history of the true roll angle and the estimated roll angle on the same graph.
- Using the second implementation, plot a time history of the true roll angle and the estimated roll angle on the same graph.
- Comment on whether the second implementation reduces drift.

References


